The inadequacy of first-order treatment wetland models

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Abstract

First-order models are in common use for design of treatment wetlands. These are frequently based on input/output (I/O) data; less frequently, on internal longitudinal transect data. The presumption is often made that the parameters of these models, i.e. the rate constants, are true constants and do not depend on factors such as hydraulic loading rate and inlet concentration. Another common presumption is that plug flow is a reasonable approximation to the hydraulic conditions in the wetland. This paper assembles a test wetland simulation, based on known information about vegetation resistance, treatment effects of vegetation, and residence time distributions. The test wetland is then used to provide simulations of different experimental and design protocols, such as transect measurements and I/O data from parallel and sequential detention time studies. Those simulations are shown to be consistent with real system data, thus confirming the data foundation of the test wetland behavior. The test wetland results, in common with the observations for nearly all treatment wetlands, show declining concentrations that approach a plateau at long detention times. The synthetic ‘data’ so produced is free from the stochastic vagaries of real wetlands, and thus provides a platform for understanding the deterministic component of behavior. The several variations of the plug flow model were then fit to the ‘data’, with generally excellent correlation coefficients ($R^2 > 0.95$). However, the parameters (rate constants and apparent background concentrations) were found to be very strong functions of hydraulic loading and inlet concentration. This variability renders the models incapable of acceptable performance in design. Addition of a third parameter, such as a dispersion number, does not solve the inherent problems; nor does the retreat to loading regressions. It is suggested that new paradigms are needed that incorporate the ability to describe short-circuiting and spatial distributions of vegetation. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Previous literature suggests first-order, irreversible pollutant reduction removal models for treatment wetlands. First-order models may be either area specific, and thus determine the necessary wetland acreage (Schierup et al., 1990; Mitsch et al., 1995; Upton and Green, 1995; Kadlec and Knight, 1996), or volume specific, and thus determine the wetland water volume (USEPA, 1988). These paradigms are deterministic, meaning that the equations purport to repre-
sent the wetland output concentrations in response to inlet concentrations, flow rate, and area or volume. At this point in the evolution of treatment wetland technology, only simple models can be calibrated for most operational systems.

The published models presume constancy of parameters, such as the rate constant, for the description of any individual wetland. It is to be expected that performance also includes a good measure of variability that is not predicted by the average values of these forcing variables. That variability is caused by unpredictable events, such as the fluctuations in input flows and concentrations, and by changes in internal storages, as well as by weather, animal activity and other ecosystem factors. These stochastic phenomena introduce scatter into a treatment wetland data set.

There are two hydraulic loading effects that can have very important consequences for data interpretation and design. Firstly, the amount of water entering the wetland from the source may vary, contributing to inlet hydraulic loading effects. Inlet hydraulic loading is defined as the rainfall equivalent of the added wastewater:

\[ q_i = \frac{Q_i}{A} \]

where \( A \) is the wetland planar area (m\(^2\)), \( q_i \) is the inlet hydraulic loading rate (m/day) and \( Q_i \) the inlet water flow rate (m\(^3\)/day).

In the absence of other effects, an exponential model results for steady conditions, but parameters may depend upon hydraulic loading. Secondly, water entering or leaving the wetland via rain (precipitation, \( P \); m/day) and evapotranspiration (ET; m/day) produces secondary hydraulic loadings. The exponential model is then theoretically invalid, and parameters may additionally depend upon hydraulic loading.

There are many potential effects of changed loading, or the changed linear velocities that result from it. Detention times change with loading. Mixing and diffusional mass transfer are velocity dependent. Short-circuiting and other attributes of the flow patterns may depend on average water speed. In friction-controlled wetlands, depth changes with hydraulic loading. Chemical uptake may be velocity dependent.

Input/output (I/O) data on flows and concentrations may be used for calibration, producing model parameters that are here designated as the I/O parameters. Alternatively, data collected along the length of the treatment wetland may be used for parameter estimation, producing an alternative parameter set, here called the transect parameters.

Expanding inter- and intrasystem databases provide the opportunity to examine the assumptions inherent in current models. This paper examines the current design frameworks, to provide further insights on their uses and limitations.

2. Current model structures

For both subsurface flow (SSF) and free water surface (FWS) wetlands, the water mass balance is:

\[ Q = Q_i + Wx(P - ET) = uWh \]

where ET is evapotranspiration (m/day), \( h \) is free water depth (m), \( P \) is precipitation (m/day), \( Q \) is the water flow rate (m\(^3\)/day), \( Q_i \) the inlet water flow rate (m\(^3\)/day), \( u \) the superficial velocity (m/day), \( W \) the wetland width (m) and \( x \) is the distance from the inlet (m).

Design often contemplates a stable period of operation, with constant flows \((P - ET = 0)\).

The rate equations are presumed to be of the form:

\[ J = k(C - C^*) \]

\[ R = k_v(C - C^*) \]

where \( C \) is the concentration (g/m\(^3\)), \( C^* \) the apparent background concentration (g/m\(^3\)), \( J \) the areal removal rate (g/m\(^2\)/year), \( k \) the areal removal rate constant (m/year), \( k_v \) the volumetric removal rate constant (1/day) and \( R \) the volumetric removal rate (g/m\(^3\)/day).

The two alternate rate constants are related by the free water depth, with \( k = (\varepsilon h)k_v \), where \( \varepsilon \) is the wetland void fraction, usually together with a change in time scale from years to days. The rates of Eqs. (3) and (4) are used in combination with the water mass balance (Eq. (2)) to obtain pollu-
tant concentration profiles. If flow is nondisper-
sive, the designation is for a plug flow reactor
(PFR). With constant \( P = ET \), exponential
profiles are predicted (reaching a plateau of \( C = C^* \)):

\[
\frac{C - C^*}{C_i - C} = \exp\left[ -\frac{ky}{q} \right] \quad (5)
\]

\[
\frac{C - C^*}{C_i - C^*} = \exp\left[ -k_v\tau y \right] \quad (6)
\]

where \( C_i \) is the inlet concentration (g/m²), \( q \) the
hydraulic loading rate (m/year), \( L \) the wetland
length (m), \( y \) the fractional distance through the
wetland (\( = x/L \)), and \( \tau \) is the nominal detention
time (day).

In the process of data fitting for treatment
wetlands, the parameter \( C^* \) is sometimes observed
to be close to zero (ammonium, nitrate, phospho-
rus), and sometimes not (organic nitrogen, TSS,
biological oxygen demand (BOD)). A distinc-
tion must be made between true background con-
centrations (\( C_b \)) and apparent background
concentrations (\( C^* \)). True background is the re-
sult of atmospheric and groundwater chemical
additions, chemical speciation, and the biogeo-
chemical cycle. It also reflects the wetland back-
ground hydraulics. The apparent background is a
data-fitting parameter that serves the purpose of
allowing deviations from a strict exponential de-
cline in concentration through the wetland. In
some cases, but not all, the two may be found to
be equal.

The presumption is very often made that \( C^* \),
and \( k \) or \( k_v \) are constants, except for possible
seasonal effects. Seasonality, or the associated
temperature effects, is not considered in this pa-
per. There is, however, the possibility that these
supposed constants are in fact functions of the
remaining wetland characteristics and operating
conditions:

\[
k = \psi_k(h,q,C,\mathcal{D},P-ET) \quad (7)
\]

\[
C^* = \psi_C(h,q,C,\mathcal{D},P-ET) \quad (8)
\]

where \( \mathcal{D} \) is the wetland dispersion coefficient (m²/
day), \( \psi_k \) the background concentration function
symbol and \( \psi_C \) the rate constant function symbol.

The following sections explore the potential
strength of these functional relations with respect
to dispersion (\( \mathcal{D} \)), hydraulic loading (\( q \)) and inlet
concentration (\( C \)). It is known that depth (\( h \)) and
net atmospheric augmentation (\( P - ET \)) have
strong effects on \( k \) and \( C^* \) (Kadlec and Knight,
1996). Depth is held constant, and there is no
atmospheric augmentation in the following
discussion.

3. Effects of nonideal flow on inert tracers

There are many tracer studies of treatment
wetlands (Fisher, 1990; Stairs, 1993; Kadlec,
1994). These display a large amount of dispersion,
as measured by the spread of a tracer impulse
upon passage through the wetland. Dispersion
numbers (\( \mathcal{D}/uL \)) are typically in the range 0.2–
0.4, which places the results in the ‘large amount
of dispersion’ category (Levenspiel, 1972). A num-
ber of mechanisms can cause dispersion, including
wind mixing and bioturbation, shear-induced tur-
bulence, and velocity profile effects. In a densely
vegetated wetland with low flow rates, the velocity
profile effects would be dominant. There are two
contributions: vertical velocity profiles, caused by
bottom friction and the free air–water interface;
and channeling of flows around vegetation,
through open micro- or macro-channels. The resi-
dence time distribution (RTD) quantifies the dis-
ctribution of detention times for a given wetland
(Kadlec, 1994).

Although well-known techniques are available
to calculate the effect of the RTD on pollutant
removal performance (Kadlec and Knight, 1996;
based on Levenspiel, 1972), these have rarely been
used in treatment wetland data analysis or design.
Instead, the steady, constant flow variant of the
first-order model has been force-fitted in data
analysis, and accepted as ‘conservative’ in design.
It has been previously recognized that nonideal
mixing can cause large errors in rate constant
estimation (Kadlec et al., 1993). If the hydraulic
efficiency of the wetland in design is less than that
in the data sets that generated the rate constants,
as indicated by more mixing or short-circuiting,
then corrections for the degree of nonideality
should be applied (Kadlec and Knight, 1996).
However, those same effects also cause drastic
failure of the assumptions of parametric constancy, with respect to hydraulic loading and inlet pollutant concentration. Model exploration of a typical example wetland serves to illustrate the character and magnitude of these effects.

4. Totally untreated short-circuiting

It is useful to examine a limiting case of poor flow distribution that is easily visualized. If a fraction \( \phi \) of the water bypasses the ‘action zones’ of the wetland without treatment, and the balance is treated in plug flow, then a simple blending calculation produces the combined output (Fig. 1):

\[
\begin{align*}
C_t - C^* &= \exp[-k/q'] = \exp[-k, \tau'] \\
C_o &= \phi C_t + (1 - \phi) C_t
\end{align*}
\]

where \( A' \) is the remaining treatment area not involved in bypass (m\(^2\)), \( C_o \) is the apparent background concentration (g/m\(^3\)), \( C_t \) is the treated concentration (g/m\(^3\)), \( q' \) the hydraulic loading to remaining area (m/year), \( T' \) the detention time in remaining volume (day) and \( \phi \) is the fraction of flow short-circuited without treatment.

Eq. (9) shows that the bypassed water causes a contribution to the exit concentration. In the limit of a very large system, corresponding to very low hydraulic loading, the treated concentration \( (C_t) \) approaches a lower limit designated as \( C^*_t \). For this limiting case, the limiting value of the blended output will be:

\[
C^* = \phi C_t + (1 - \phi) C^*_t
\]

The bypassed contribution is not a return from the wetland biogeochemical cycle, nor is it an irreducible residual passing through treatment. This contribution is a bypassed, untreated residual. It is interesting that the form of Eq. (11) is linear in \( C_t \) \( (C^* = a + bC_t) \). This form has been retrieved by data analysis in Kadlec and Knight (1996); for instance, for BOD, they report \( C^* = 3.5 + 0.053C_t \).

This simple example is an extreme case. Because the short-circuited water receives no treatment, this situation is termed untreated short-circuiting. Most wetlands would be expected to have a spectrum of flow paths, along which water would move at different speeds. The wetland RTD quantifies this spectrum.

5. Treated short-circuiting

A typical treatment wetland RTD is a bell-shaped continuous function with an elongated tail (Fig. 2). The resulting cumulative distribution is an ‘s’-shaped curve. These may be parameterized with either a tanks-in-series (TIS) or plug flow with dispersion (PFD) model; and, with
a rate equation, used to predict outlet concentrations of pollutants (Kadlec and Knight, 1996). The PFD model is explored here. The assumption of a spatially uniform rate constant is made. Therefore, although some water moves faster than other water, there is some degree of treatment for even the fastest flowpaths. For the constant depth scenarios under consideration, the sole modifier of removal is then the travel time.

A well-known set of mathematical manipulations, including integration, produces results from the continuum theory (Levenspiel, 1972). The pollutant mass balance equation includes spatial variability:

\[ \frac{1}{Pe} \frac{\partial^2 C}{\partial y^2} - \frac{\partial C}{\partial y} - Da \cdot C = 0 \]  

(12)

where \( Da \) is the Damköhler number, \( Da = k_v \tau = k_v L/u \), and \( Pe = uL/D \) = \( 1/\tau \).

The appropriate wetland boundary conditions for this mass balance are known as the closed–closed boundary conditions (Fogler, 1992):

at \( y = 0 \) \[ [C]_{y=0} - [C]_{y=0+} = \left[ -\frac{1}{Pe} \frac{\partial C}{\partial y} \right]_{y=0+} \]  

(13)

at \( y = 1 \) \[ \left[ \frac{\partial C}{\partial y} \right]_{y=1} = 0 \]  

(14)

These imply that no material can diffuse back from the wetland into the inlet pipe, nor back up the exit structure at the wetland outlet. These are different from the open–open boundary conditions that are appropriate for river studies. There are no closed form solutions to the former case, but numerical solutions to the closed–closed tracer mass balance have been available for more than two decades (Levenspiel, 1972).

This second-order ordinary differential equation was first solved by Danckwerts in 1953 (Fogler, 1992). Because of the linearity of the model, it is also applicable for \((C - C^*)\) replacing \((C)\):

\[ \frac{C_o - C^*}{C_i - C^*} = \frac{4b \exp\left(\frac{Pe}{2}\right)}{(1 + b)^2 \exp\left(\frac{bPe}{2}\right) - (1 - b)^2 \exp\left(\frac{-bPe}{2}\right)} \]  

(15)

\[ b = \sqrt{1 + \frac{4Da}{Pe}} \]  

(16)

Inspection of Eq. (6) shows that the effect of doubling the length is the same as halving the velocity, because both double the exponent, \( k_v L/u = k_v \tau \) (Damköhler number). Therefore, there is interchangeability of detention time caused by velocity changes, and caused by changing distance down the wetland. However, inspection of Eqs. (12) and (13) shows a more complicated relation for velocity and length effects, and the interchangeability is no longer necessarily valid.

For \( D = \) constant, there is no interchangeability, and there is an effect of wetland aspect ratio \((L/W)\), with longer, narrower wetlands producing more removal. However, for \( Pe = \) constant, interchangeability remains, and there is no effect of aspect ratio. The later situation prevails for porous media (Levenspiel, 1972), and may therefore prevail for flow in litter, stems, or gravel media. FWS wetland tracer data support the concept of \( Pe = \) constant (Werner, 1996).

This formulation produces concentration distributions that are very nearly exponential in shape, but different from the PFR case. Suppose \( C_i = 100 \text{ g/m}^3, C_b = 5 \text{ g/m}^3, Pe = 2.75 \) and \( Da = 1.75 \) \((k = 22.2 \text{ m/year}, \sigma_h = 0.25 \text{ m, } k_v = 0.243 \text{ day}^{-1}, \tau = 7.21 \text{ days})\). (These values will be generated in a later section.) The concentration profile may be generated via Eqs. (15) and (16), and then fit with the PFR version of the \( k - C^* \) model. The result is an excellent fit:

Case 1 Fit of the flow-weighted ‘data’: \( k = 13.7 \text{ m/year} \quad C^* = 25.1 \text{ g/m}^3 \quad R^2 = 1.000 \)

In effect, the flow nonideality has been swallowed up into the more simple PFR model parameters. The effect is not minor: the apparent rate constant is 38% lower, and the background concentration is five times larger than if mixing
were accounted for. Therefore, wetland factors that affect mixing will also affect the (force-fitted) PFR model parameters. Inter-system variability has thereby been increased.

6. Partially treated short-circuiting

6.1. An example wetland

The theory of the previous section is based on the presumption of a spatially uniform rate constant across the wetland. However, many treatment wetlands have spatial distributions of vegetation density, which in turn imply spatial distributions of flow resistance and treatment efficiency. Treatment would presumably be better along the slower flow paths, with dense biomass; and worse along the faster paths, with sparse biomass.

A free water surface wetland will be considered in this section. For ease of visualization, a discretized version of those manipulations will be used here (Fig. 3). This example distribution has a mean detention time of 7.21 days, and a dispersion number of $D = 0.36$ ($Pe = 2.75$).

A high-velocity path results from lack of vegetative resistance; a low-velocity path from large vegetative resistance. The hydraulic friction relation that describes this relation has been set forth (Shih and Rahi, 1982; Hokosawa and Horie, 1992), and for laminar flow is given by:

$$u = \alpha' S/X = \alpha X$$  

(17)

where $\alpha$ and $\alpha'$ are the proportionality constants (day$^{-1}$), $X$ the vegetation surface area density ($m^2/m^3$) and $S$ the slope of the water surface (m/m).

This relation states that, for a specified overall water slope, the water velocity is inversely proportional to the immersed vegetation surface area. Barrett (1996) has shown that the wetland velocity field, and hence the RTD, is dependent on the spatial distribution of vegetative resistance.

The wetland biomass density influences many wetland pollutant removal processes. This may be due to the biofilms that coat submerged litter and stems (USEPA, 1988; Water Pollution Control Federation, 1990; Polprasert et al., 1998), or to the biogeochemical cycling that creates net uptake to new sediments (Kadlec, 1997). Here, it is presumed that the rate constant is proportional to $X$:

$$k = (\alpha h) k_v = \beta X$$  

(18)

where $\beta$ is the proportionality constant ($m^2$/day).

The effect of spatial patterns of vegetation density is to create spatial patterns of the removal rate constant. The effect on pollutant removal is thus twofold: flows through low vegetation density regions are fast and ineffective, while flows through high vegetation density regions are slow and effective.

To conceptualize these effects, the spatial arrangement of vegetation will be considered to be uniform but unequal densities on parallel paths of uniform depth (Fig. 4). The vegetation density for each flow path was set so that the cumulative RTD of Fig. 3 was achieved, which is equivalent to setting the volumetric flow along each of the five paths. The width of each flow path was determined from the velocity via Eq. (17) with $\alpha = 1.0$ day$^{-1}$, and the path volumetric flow rate. This produces wider flow paths for the slower and smaller volumetric flows. If one were to walk across this wetland, most of the traverse would be in areas moving slower than the mean velocity.

![Example Wetland](image-url)
For instance, 62% of the width is associated with the slowest 35% of the volume flow.

Pollutant removal along each flow path, for a hypothetical constituent, is computed from Eq. (5) (equivalent to Eq. (6) for constant depth), with a rate constant determined by Eq. (18). A true wetland background concentration of $C_b = 5 \text{ g/m}^3$ is assigned. The area mean vegetation relative density is $X = 5.33$, which would result in an area-mean $k = 22.2 \text{ m/year} (\beta = 4.16 \text{ m}^2/\text{day})$.

This corresponds to $k_v = 0.243 \text{ day}^{-1}$. The entering concentration is chosen to be 100 g/m$^3$ for simplicity.

The results of this example show greatly disparate concentration reductions along the individual flow paths (Figs. 5 and 6). Outlet concentrations range from 5 to 87.4 g/m$^3$, with a flow weighted mean of 48.8 g/m$^3$ (Fig. 5). This is
the concentration that would be measured if the entire flow were sampled at a common outlet weir. However, if the exit region were sampled by collecting spatially uniform samples from the interior of the wetland, the result would be a spatial mean concentration of 31.4 g/m³. This difference in the two measurement techniques has long been recognized in the literature. Levenspiel (1972) refers to the flow-weighted concentration as the ‘mixing cup’ value, and refers to the spatial mean as the ‘through the wall’ value for chemical reactors with dispersion.

Both measures produce longitudinal concentration profiles that curve less than the PFR profile for the mean wetland conditions (Fig. 7), and appear to tend toward asymptotes well above the intrinsic background of 5 g/m³.

7. Retrofitting the example wetland ‘data’

The results of this numerical example may next be treated as ‘data’ from a functioning treatment wetland. First, suppose that the water sampled in such a way that the results could be flow-weighted at each interior sample point. This would be very difficult in the field, because flow measurements would need to accompany each sample. The result would be the top curve in Fig. 7. Because of the apparent asymptote, the PFR $k - C^*$ model is better able to fit the data, and was used for analysis. A second, field-easier sampling method is to acquire data along equally spaced longitudinal transect points, thus providing spatially uniform sampling. This technique has been used at several treatment wetlands, including Boney Marsh (Davis, 1981) and Benton, KY (Tennessee Valley Authority, 1990).

The results of model ‘data’ fitting for 10 sections of the wetland were:

Case 2 Fit of the flow-weighted ‘data’: $k = 28.8$ m/year $C^* = 44.3$ g/m³ $R^2 = 0.997$

Case 3 Fit of the spatial ‘data’: $k = 47.0$ m/year $C^* = 31.9$ g/m³ $R^2 = 0.996$

Both fits are excellent representations of the ‘data’, but neither are close to the intrinsic parameters, especially for $C^*$. As discussed in the short-circuiting section, a portion of the incoming water is being passed rapidly to the outlet, without significant treatment, creating the illusion of a high background concentration. The character of this result is not altered if there is a zero intrinsic background concentration; only the fitted $C^*$ is reduced.

7.1. Supporting evidence

The South Florida Water Management District conducted detailed studies of Boney marsh in southern Florida for more than 11 years. Both I/O and transect data were acquired for this 49 ha, constructed phosphorus treatment wetland (Davis, 1981; Mierau and Trimble, 1988). For illustration, monthly data from two parallel transects was averaged to provide the mean behavior for March 1978–March 1979 (Fig. 8). Long-term averaging is required to avoid the synoptic error, occasioned by the detention time lags inherent in any single transect; as well as stochastic variability. For illustration, the two parallel transects from one sampling date show large scatter (Fig. 8).

The Boney transects show the characteristics of the closed–closed boundary conditions: a sudden drop in concentration as water passes into the system, and a plateau at the exit end (Eqs. (13) and (14)).
Fig. 8. Average concentration profiles, I/O data, and model for Boney Marsh.

The $k - C^*$ model fits the transect data very well: $k = 29.4$ m/year, $C^* = 12.9$ mg/m$^3$, $R^2 = 0.992$. These parameter values do not agree with those developed from I/O data for that same year: $k = 16.9$ m/year, $C^* = 11.3$ mg/m$^3$. The transect $k$ value is not in the range developed from I/O data, $k = 7.7$–21.0 m/year, $N = 11$. The transect $C^*$ value is in the range developed from I/O data, $C^* = 10.1$–18.9 mg/m$^3$, $N = 11$.

There is a 27% difference in concentration between the long-term average of exit transect concentration of 12.9 mg/m$^3$ (spatial) and the long-term exit weir concentration of 16.4 mg/m$^3$ (flow weighted). This agrees with the concept developed from the example wetland ‘data’, as shown in Fig. 5.

8. Compartmentalization

There is another possibility for field operation of essentially the same example wetland, in which the water is physically collected at the end of a subsection, mixed and redistributed into the next section. This technique was used at the Listowel facility ($N = 5$; Herskowitz, 1986) and the Arcata facility ($N = 8$; Gearhart et al., 1989). Accordingly, this option was explored for the example wetland, for $N = 10$ equal subsections. Regression of the ‘data’ from this numerical experiment produced:

Case 4 Fit of the flow-weighted ‘data’: $k = 18.7$ m/year, $C^* = 5.0$ g/m$^3$, $R^2 = 1.000$

In other words, this compartmentalization drove the system to perform in the plug flow mode. The small difference in the retrieved rate constant (18.7 versus 22.2) is due to fitting the average operation rather than fitting the average vegetation density.

8.1. Supporting evidence

Gearhart et al. (1989) report performance of compartmentalized wetlands. BOD reduction was better in a wetland with eight units (86%) than in a wetland with two units (71%).

9. Effects of hydraulic loading

9.1. Corresponding data sets

The preceding section addresses the issues of concentration profiles and performance for fixed hydraulic loading. However, it is equally important to assess effects of changing total detention time ($\tau$), or equivalently, changing areal hydraulic loading rate ($q$). As indicated earlier, both the PFR and PFD (constant $Pe$) models are invariant with respect to changes in loading. In other words, what is seen at the wetland midpoint would be seen at the wetland outlet for a doubled flow rate.

There are therefore two ways to explore the effects of hydraulic loading. One is to observe the performance of a shorter wetland under the same inflow. This is of course accomplished by observing the flow-weighted concentrations at various points along the length of the wetland. This is Case 2, dealt with in the preceding section.

The second is to observe the performance of the entire wetland under different inflow conditions. The example wetland was subjected to a variety of inflows ($Q$), with other conditions the same as previously described. The span of the flow rate changes was selected to produce the same range as the selection of intermediate distances (0–7.21 days detention). The results were identical to Case 2, thus indicating that distance and velocity are interchangeable, even in the more generalized sce-
nario of spatially variable vegetation resistance and spatially variable rate constants.

9.2. One parameter fits

It has been shown that data which display a true background concentration will lead to a hydraulic loading dependence of the PFR rate constant if it is presumed that $C^* = 0$ (Kadlec and Knight, 1996). This is illustrated in Fig. 9, in which the example wetland ‘data’ are used to determine $k_1$ under the assumption of $C^* = 0$. The lower curve in Fig. 9A is fit to the inlet concentration and the value 10% of the way down the wetland, and yields $k_1 = 17.7$ m/year. The upper curve in Fig. 9A is fit to the inlet concentration and the value 100% of the way down the wetland, and yields $k_1 = 9.1$ m/year. The hydraulic loading to the 10% point is 0.347 m/day; to the 100% point 0.0347 m/day. Over the length of the system, the change in $k_1$ is nonlinear. Fig. 9B shows the resultant dependence of $k_1$ on hydraulic loading.

Because of the interchangeability of velocity and length for the example wetland, the same result will occur for low and high flows to the example wetland.

9.2.1. Supporting evidence

Transect BOD data was collected biweekly from Listowel system 4, and averaged over a calendar quarter, thus avoiding synoptic error and stochastic masking (Herskowitz, 1986). This five-unit, compartmentalized wetland data displays the effect found for the example wetland (Fig. 10).

9.3. Parallel data sets

Loading changes may be accomplished in two ways: via replicated wetland cells receiving different flows; or via sequential operation of the same wetland cell at different flows. The former has the advantage of identical water quality and meteorology; the latter has the advantage of a single wetland environment. The time factor favors multiple cells in parallel. Reasonable replication has been achieved in practice (Kuehn and Moore, 1995). The question of effects of different loadings was addressed here via the model of the example wetland. The model was run for a range of hydraulic loadings, including the base case flow rate. Parameters $k$ and $C^*$ were determined by least-

![Fig. 9. Transect data fitting for the example wetland. (A) I/O fits for short and long wetland points. (B) Resultant load dependence of the one-parameter rate constant.](image)

![Fig. 10. Measured load dependence of the one-parameter rate constant for Listowel wetland channel number 4.](image)
squares fitting. The correlation coefficients for fits of I/O data were all greater than $R^2 = 0.999$.

Both $k$ and $C^*$ were found to increase strongly with hydraulic loading rate (Fig. 11). The spectrum of rate constants includes the value 22.2 m/year, which characterizes the data-generating wetland. But the values of $C^*$ are all higher than the true value of 5 g/m$^3$, which characterizes the data-generating wetland.

9.3.1. Supporting evidence

Data was collected at Gustine, CA for five parallel wetlands, over a 1-year period, at three different hydraulic loading rates (Walker and Walker, 1990; reiterated in the North American Database, USEPA, 1994). The BOD inlet concentrations varied, allowing a $k - C^*$ regression for each wetland separately. The $k$ values increase from 12 to 52 m/year as hydraulic loading increases from 1 to 4.2 cm/day (Fig. 12). Thus, this parallel cell data, from one site with one influent, displays the same characteristics as the example wetland.

A second, larger data set has been collected from 54 treatment wetlands in Denmark (Schierup et al., 1990; reiterated in Kadlec and Knight, 1996). These are soil-based reed beds, most treating primary domestic effluent. Only values of $k$, may be extracted from this data, because of the limited amount of data for each system. Despite differences in wetland geometry, soils, and other factors, this intra-system data shows a very strong relation between $k$ and hydraulic loading (Fig. 13).

10. Effects of inlet concentration

The example wetland model was fed a series of different inlet concentrations, which produced transect ‘data’ sets. Those were then fit with the $k - C^*$ model. The simulation results show no effect on the $k$ value, as was the case for untreated short-circuiting ($k = 28.8$ m/year). However, as for untreated short-circuiting, there is a strong dependence of the $C^*$ value on the inlet concentration: $C^* = 2.9 + 0.41C_i$ ($R^2 = 1.000$).
This effect is reported for domestic wastewater treatment for BOD in wetlands (Kadlec and Knight, 1996), as noted earlier.

11. Expanded models

The foregoing has illustrated that one- and two-parameter PFR models are not very successful in maintaining parametric constancy across varying operating or design conditions. Two potential ‘fixes’ seem obvious: add a third parameter, in the form of a loading dependence of the rate constant; or add a third parameter, in the form of the dispersion coefficient, or another equivalent mixing parameter.

In the first instance, the rate constant is written as:

\[ k = \frac{k'}{q^m} \quad \text{or} \quad k_v = \frac{k'_v}{q^m} \]  

(19,20)

where \( m \) is a parameter to describe the load dependence. For the steady constant flow situation, this produces:

\[ \frac{C - C^*}{C_i - C^*} = \exp\left[ -k'y/q^{1-m} \right] = \exp\left[ -\varepsilon h k'_v \tau^{1-m} \right] \]  

(21,22)

This loading formulation has been used to describe overland flow treatment, a close cousin to wetland treatment (Smith and Schroeder, 1985; reiterated in Water Pollution Control Federation, 1990).

Preliminary testing shows that addition of this third parameter \( m \) improves the fit of the ‘data’ from the example wetland \( R^2 = 1.000 \), but does not remove the loading effect on \( C^* \). It was found that \( C^* = 8.75q - 0.73 \) \( R^2 = 0.990 \), where \( q \) is the hydraulic loading rate (cm/day).

The second potential strategy is to include dispersion in the model, i.e. use Eq. (15). Barrett (1996) found that reasonable approximations to wetland tracer responses could be obtained from the two-dimensional velocity field coupled with dispersion, so it seems plausible that this third parameter would improve the fit of data. Accordingly, the measured value of \( Pe = 2.75 \), obtained from the tracer response ‘data’ for the example wetland, was selected, and Eq. (15) fit to the example wetland concentration ‘data.’ Excellent fits were obtained, with \( R^2 \geq 0.999 \). However, both \( k \) and \( C^* \) remained strongly correlated with hydraulic loading rate: \( k = 2.7q + 7.8 \) \( R^2 = 0.983 \), \( C^* = 3.3q + 16.7 \) \( R^2 = 0.913 \).

12. Summary and conclusions

An example treatment wetland has been postulated, with internal flow patterns that correspond to the RTDs measured in real systems. A spatial distribution of biomass has been inferred from this RTD, and known and measured effects of biomass on water velocity. Pollutant reduction in example wetland has been postulated to also depend upon biomass, and to approach a non-zero background. All of these attributes are based upon existing scientific studies. The response of this hypothetical wetland was then numerically explored, over the space of longitudinal distance, inlet flow rate, and inlet concentration. These simulations produced concentration profiles, and the associated I/O data, for the example wetland. Such simulations are to be regarded as the mean performance over long-term data acquisition, which eliminates stochastic effects and synoptic error. Depth effects and atmospheric inputs are known to be significant, but are not considered here.

Existing design models were then calibrated to this synthetic ‘data.’ One-parameter \( k \), two-parameter \( C^* \), and three-parameter \( (\varepsilon \) or \( Pe, m) \) PFR models were fit, and were found to produce good to excellent fits of the synthetic data. The universally observed decline (all models) to a plateau \( (C^* = 0) \) is reproduced. However, none of these models returned the parameter values that were used to construct the synthetic data. Large values of \( C^* \) were obtained, which are clearly attributable to treated or untreated short-circuiting water. If the flow pattern is forced toward plug flow, by compartmentalization with redistribution, then calibration tended toward producing the parameter values that were used to construct the synthetic data.

Significant differences in transect results can occur due to the disproportionate fraction of the
wetland surface that is associated with slow moving, well-treated water. Flow-weighted and area proportional sampling methods are not equivalent.

In addition to calibration accuracy, a good design model should contain parameters that are invariant with respect to operating conditions. None of the models passed this test. The parameters were found to depend strongly on hydraulic loading ($k$ and $C^*$), inlet concentration ($C^*$). These effects are worst for one-parameter models ($C^* = 0$), but are not absent even for the three-parameter models. These features have been demonstrated to exist in the operational data from many treatment wetlands, as already documented. This indicates that the simulation results are not the artifact of a poorly chosen example wetland.

The implications for process modeling are very serious. The results indicate that PFR models are valid only for the exact data sets that produced them (first sets), and not for other data sets (second sets), even if the second set is within the range of the first. The manifestations of this are most notably extremely variable $k$ and $C^*$, even for the description of a single wetland. The assertion of a single value for $k$ has been demonstrated to be false. There is a spectrum of values, even from one wetland. At most, the central tendencies may be characteristic.

The assertion of $C^* = 0$ has been shown to be false. Even if there is no irreducible component of the pollutant, or no return flux from the ecosystem, nonideal hydraulics will create the appearance of a background concentration.

The fact that $C^*$ depends on both inlet concentration and hydraulic loading means that, in effect, there is ‘floating’ background concentration. Real wetlands with partially untreated partial short-circuiting will pass a partially treated fraction of the incoming concentration to the exit, and that partial treatment worsens as the velocity (hydraulic loading) goes up.

Just as serious is the loading effect on the rate at which the concentration approaches this floating background level. Consider the Danish data in Fig. 13, which represent the one-parameter ($C^* = O$) model. (This is chosen to avoid confusion about the $C^*$ effect.) Faced with a design calculation with 100 g/m$^3$ inlet, one would find that detention times between 1 and 28 days would produce virtually unchanging outlet concentrations: $C_o = 17.0 \pm 2.1$ g/m$^3$. This range of values is clearly well within the intrasystem stochastic variability (Kadlec and Knight, 1996). The conclusion is that any Danish wetland, $1 < \tau < 28$ days, will produce about 83% reduction, and there is no operational strategy that can improve upon it.

Adding a third parameter ($D$ or $Pe$, $m$) can correct for the dispersion portion of the nonideal behavior, but not the degree of treatment afforded to the short-circuited water. This belies the intuitive presumption that wetlands with better hydraulic characteristics will compensate for PFR analysis of data. This will be true only if there is treatment along the fast paths. In other words, if short-circuits are due to topography, and occur through dense biomass, then this intuitive idea is correct; but if short-circuits are due to sparse biomass, then that intuition may be wrong.

It is intuitively appealing to use aspect ratio as a potential control on short-circuiting. However, Kadlec and Knight (1996) show that the wetland RTD is insensitive to aspect ratio; both long and, and short and wide, wetlands display the same general distribution of detention times. The effect of aspect ratio would manifest itself in the Peclet number, which characterizes the dispersion, length and velocity combination for a specific wetland. Kadlec (1994) found a fairly narrow band of values for several wetlands. Further confirmation is provided by Werner (1996), who found only small effects of Peclet number for 49 wetland RTDs.

Side-by-side treatment wetland data do support an advantage for high aspect ratio, in terms of $k$ values. This may be due to a linear velocity effect, or to a better gross areal efficiency. For instance, two sets of side-by-side wetlands at Listowel showed higher $k$ values for nitrogen species and BOD, but not for phosphorus, in the longer and narrower wetlands (Herskowitz, 1986). Similarly, Knight et al. (1994) tested three aspect ratios in side-by-side experiments, and found higher $k$ values at higher aspect ratio for nitrogen species, TSS and BOD, but not for phosphorus.
However, the increased efficiency does not imply immunity from RTD effects. Although they have increased $k$ values, the long and narrow systems are still subject to the same deficiencies that have been explored in this paper: neither the fitted rate constants ($k$) nor the fitted background concentrations ($C^*$) are invariant with respect to hydraulic loading and inlet concentration.

Loading correlations (Kadlec and Knight, 1996), which plot outlet concentration (g/m$^3$) versus pollutant loading (g/m$^2$ day), suffer from the same parametric inconsistency. Variation of hydraulic loading creates one type of effect, via $k$ and $C^*$, and flow patterns; while variations in inlet concentration creates another effect.

Thus, it appears that none of the commonly accepted, simple, wetland treatment performance models are capable of consistent descriptions with invariant parameters. A large part of the intersystem variability that is observed in rate constants and background concentrations appears to be based in a fundamental inadequacy of the simplistic models now in use.

On a more positive note, the compartmentalization of the wetland appears to hold promise for driving the actual performance toward situations that are more efficient and describable by PFR models with minimal parametric variability. That compartmentalization can be accomplished by collecting (and therefore blending) the water from one cell and then redistributing it into the next. For less cost, but perhaps with less efficiency, the compartmentalization may be accomplished with deep zones transverse to flow (Knight et al., 1994). Alternatively, a system of baffles, coupled with strategic spatial placement of vegetation, may increase the treatment to a limited extent (Barrett, 1996). In any case, it appears necessary to intercept, treat, and disrupt the water that tends to travel along higher speed paths to the outlet.

It is suggested that models which contain more internal detail than is provided by I/O measurements guide the acquisition of more treatment wetland data. A better understanding of the details of internal hydraulics appears essential for the furtherance of treatment wetland design modeling.

References


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