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Dynamic Properties of Granulated Rubber/Sand Mixtures


ABSTRACT: Processed waste tires mixed with soils are applicable in lightweight fills for slopes, retaining walls, and embankments that may be subjected to seismic loads. Rubber’s high damping capacity permits consideration of granulated rubber/soil mixtures as part of a damping system to reduce vibration. The dynamic properties of granulated rubber/soil mixtures are essential for the design of such systems. This research investigates the shear modulus and damping ratio of granulated rubber/sand mixtures using a torsional resonant column. Specimens were constructed using different percentages of granulated tire rubber and Ottawa sand at several different percentages. The maximum shear modulus and minimum damping ratio are presented with the percentage of granulated rubber. It is shown that reference strain can be used to normalize the shear modulus into a less scattered band for granulated rubber/sand mixtures. The normalized shear modulus reduction for 50% granulated rubber (by volume) is close to a typical saturated cohesive soil. Empirical estimation of maximum shear modulus of soil/rubber mixtures can be achieved by treating the volume of rubber as voids.

KEYWORDS: dynamic property, shear modulus, damping ratio, tire chip, waste tire, rubber, mixture, reference strain, resonant column test, Ottawa sand

Nomenclature

\begin{align*}
A & \quad \text{Dimensionless elastic stiffness parameter} \\
a & \quad \text{Soil constant used for normalized shear modulus versus shear strain relation} \\
b & \quad \text{Soil constant used for normalized shear modulus versus shear strain relation} \\
D_{\text{min}} & \quad \text{Minimum damping ratio} \\
e & \quad \text{Void ratio} \\
G_{\text{max}} & \quad \text{Maximum tangent shear modulus} \\
k & \quad \text{Dimensionless parameter related to soil plasticity index} \\
n & \quad \text{Dimensionless elastic parameter} \\
OCR & \quad \text{Overconsolidation ratio} \\
p_{\text{at}} & \quad \text{Atmospheric pressure} \\
\gamma & \quad \text{Shear strain} \\
\gamma_{r} & \quad \text{Reference strain} \\
\sigma_{0} & \quad \text{Effective mean principal stress} \\
\tau_{\text{max}} & \quad \text{Shear stress at failure}
\end{align*}

Recycled tires continue to see increasing application in engineered fills and other earthwork and paving projects. The state of the recycled tires ranges from whole tires to ground rubber. Terminology for different variations of recycled tires has been defined in ASTM (D 6270-98), and is based primarily on size and how the tire has been processed. Recycled tires are usually mixed with other materials such as soil or asphalt to meet specific needs for engineering applications. For example, they are used in asphalt paving products such as asphalt-rubber hot mix, rubber-modified asphalt, or surface/interlayer treatments (Liang and Lee 1996; Maupin 1996; Eleazer and Barlaz 1992). In geotechnical engineering, they are used in lightweight fills, backfill materials, highway embankments, soil reinforcement, and soil-retaining walls (Humphrey et al. 1993; Upton and Machan 1993; Ahmed and Lovell 1993; Lee et al. 1999). The use can be cost-effective and reduce some environmental problems from waste tires.

Many researchers have assessed some fundamental engineering properties of rubber/soil mixtures, such as compaction characteristics, compressibility, permeability, shear strength, modulus of elasticity, and Poisson’s ratio (Humphrey and Manion 1992; Edil and Bosscher 1994; Masad et al. 1996; Lee et al. 1999). However, in current practice the analysis of tire/soil mixtures is mainly for static loading, so that studies have emphasized static stress-strain behavior or rubber-soil interface behavior, with particular emphasis on the failure condition, rather than low strain behavior. The authors were unsuccessful in finding evidence of any studies on the dynamic properties of rubber/soil mixtures. Further investigation of the dynamic characteristics of rubber/soil mixtures will be beneficial to dynamic analyses, such as earthquake analysis of rubber/soil fills for slopes, embankments, and retaining walls. Such studies will also promote improved prediction of serviceability behavior under normal loads. The use of rubber/soil mixtures as a geomaterial is also very promising for vibration reduction due to rubber’s high damping behavior. Rubber/soil mixtures may be considered as part of a vibration damping system for machine foundations or railroad track beds. Dynamic properties of such mixtures is crucial for these applications.

Utilization of waste materials in construction earthwork will continue to increase and demand engineering insight for modeling behavior of earth structures formed of such materials. One aspect of this study is the development of mixture rules to assist in predicting the variation of fundamental properties with the ratio of one particulate material to another. The variation of maximum shear modulus (\(G_{\text{max}}\)) with percent rubber, as examined herein, provides insight to how mixture rules may be developed for particulate mechanisms of earth structures that include non-earth materials.

In this study, a torsional resonant column apparatus was used to obtain the shear modulus and damping. The samples were mixed to
several different percentages of rubber and sand. The maximum shear modulus and minimum damping ratio are presented with the percentage of granulated rubber. Reference strain was used to normalize a set of shear modulus data successfully. The test results were also compared to an empirical equation of maximum shear modulus assuming volume of rubber as voids. These results show that mixtures of a specific soil and granulated rubber can be modeled using conventional soil mechanics principles with a few modifications.

Properties of Granulated Rubber and Soil Used

The granulated rubber used in this study was composed of waste tires that had been mechanically shredded using multiple shredding steps and sieved to be of approximately uniform size. The sand used in the study was commercially available Ottawa test sand. Both materials were subjected to further sizing before being used in the study.

The specific gravity of shredded tire generally ranges from 1.00 to 1.36 depending on metal content (Edil and Bosscher 1994; Lee et al. 1999). The specific gravity of the granulated rubber used in this research was found to be 1.11 at a temperature of 20°C following ASTM Test Method for Specific Gravity of Soils (D 854). It is noted that the coefficient of thermal expansion of tire rubber is high relative to soil solids, so the specific gravity is more temperature-dependent than for soil. The granulated rubber was sieved to pass the standard sieve No. 4 and retained on No. 10. Therefore, the granulated rubber used for this study was of uniform gradation. The Ottawa sand was sieved to pass the No. 20 and be retained on the No. 30. The specific gravity of the Ottawa sand is 2.67.

Beatty (1981) indicated that tire rubber has high damping capacity due to its extremely high elasticity and good fatigue properties. Many characteristics of tire rubber are very different from structural and soil materials. The differences include: (1) elastic deformability is orders of magnitude greater, (2) strength and modulus of the solid particles is much lower, (3) absence of a yield point in the stress-strain curve, and (4) recovery from large deformation when stress is removed. The significant difference between rubber and soil solids complicates the behavior of the rubber/soil mixtures. The engineering properties of rubber also varies with temperature, aging/relaxation, and other environmental factors. These factors are not considered in the findings herein.

In assessing the percentage of rubber and soil solids in a specimen, the percent solids may be expressed by mass or by volume. A “by mass” percentage is more accurately measured in mixture preparation, as volume of solids depends on specific gravity, which will vary somewhat between rubber particles and with moisture content, temperature, and aging. However, deformation due to a stress change depends on the strain induced through the volume of material being considered. Percentage by volume is thus the better expression when assessing modulus and damping behavior of particulate composites.

The compressibility of rubber/soil mixtures can be relatively high, depending on packing and the rubber/soil mixing ratio (Edil and Bosscher 1994). However, the volumetric compressibility of the rubber solids is quite small. This is because the Poisson’s ratio of rubber is nearly 0.5 and varies just a little below 0.5 at large deformation (Beatty 1981). Therefore, the granulated rubber solids can be assumed to be nearly incompressible, though not undeformable. An understanding of how particulate rubber solids compress under isotropic confinement is crucial for estimating void ratio change under different confining stress. In this study, several tests were conducted on 100% particulate rubber samples under isotropic compression with careful measurement of volume change of the solid skeleton and voids to obtain volume change of rubber solids. These tests verified that the volumetric compressibility of the tire rubber solids is insignificant in the pressures applied ranging from 0 to 700 kPa. Soil solids are assumed incompressible in the stress ranges considered herein.

Sample Preparation

In determining the sample preparation method to be used, the authors sought a method that would provide a highly uniform specimen that could be representative of the state of specimens placed as fill on projects. Since most current applications of tire rubber in fills are above groundwater, the authors chose to test specimens in a dry or slightly moist state. Hardin (1961) found that sand tests in a resonant column exhibited slightly higher stiffness and lower damping when tested dry as opposed to saturated, but concluded the difference was not significant. A review of the literature did not provide any insight into the effect of dry versus saturated modulus and damping of granulated rubber, though there is not expected to be a significant difference. Study of this variable should be addressed in future studies. Testing of specimens in a dry state versus a slightly moist state could result in different behavior due to the effect of partial saturation on the effective stresses and thus stiffness of the specimens. This consideration is addressed in the results presented later.

Based on the above considerations, two sample preparation methods were adopted for construction of the approximately 7-cm diameter by 15-cm-high specimens for resonant column testing: “undercompaction” and “hand-spooning.” For undercompaction preparation of the sample, the procedure developed by Ladd (1978) was followed. The samples were sequentially compacted from eight layers of mixtures. A small amount of water was added during mixing of the samples to reduce segregation of the granulated rubber and sands. For hand-spooning preparation, the air-dried tire chips and sand were alternately and evenly spooned into the mold to achieve uniformity. The specimens were thus tested either dry or with a very low moisture content, depending on the specimen preparation technique. The samples prepared by undercompaction (UC) were qualitatively judged to appear much more uniform than samples constructed by hand-spooning (HS). For this reason, many of the specimens prepared by hand-spooning could not be considered sufficiently uniform to give valid results.

The data for the samples tested in the resonant column device are given in Table 1. The percentages of granulated rubber by volume

<table>
<thead>
<tr>
<th>% Rubber by Volume</th>
<th>Initial Void Ratio at 69 kPa</th>
<th>Preparation Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>0.45</td>
<td>UC</td>
</tr>
<tr>
<td>45</td>
<td>0.39</td>
<td>UC</td>
</tr>
<tr>
<td>49</td>
<td>0.40</td>
<td>UC</td>
</tr>
<tr>
<td>76</td>
<td>0.46</td>
<td>UC</td>
</tr>
<tr>
<td>100</td>
<td>0.35</td>
<td>UC</td>
</tr>
<tr>
<td>0</td>
<td>0.52</td>
<td>HS</td>
</tr>
<tr>
<td>27</td>
<td>0.49</td>
<td>HS</td>
</tr>
<tr>
<td>42</td>
<td>0.49</td>
<td>HS</td>
</tr>
<tr>
<td>49</td>
<td>0.45</td>
<td>HS</td>
</tr>
<tr>
<td>73</td>
<td>0.45</td>
<td>HS</td>
</tr>
<tr>
<td>0</td>
<td>0.57</td>
<td>HS</td>
</tr>
</tbody>
</table>

* UC = undercompaction. HS = hand-spooning.
were calculated from 36-h air-dried weights determined after testing. The initial void ratio is defined as the volume of voids divided by the combined volume of sand and granulated rubber solids at an isotropic confining stress of 69 kPa.

Specimen Consolidation and Resonant Column Test Results

A torsional resonant column device was used to obtain the small strain dynamic properties of the granulated rubber/sand mixtures. The resonant column theory and the description of the device used can be found in Drnevich et al. (1978). The isotropic confining pressure was progressively increased to 69, 207, 345, and 483 kPa (10, 30, 50, and 70 psi), with measurement of shear modulus and damping properties for each stage. During each isotropic consolidation stage, the granulated rubber/sand mixtures showed evidence of slow axial compression creep. The actual volume change could not be estimated since the specimens were not saturated and lateral strain measurements were not possible with the equipment. However, each consolidation stage was terminated when axial strain rate was observed to drop from a relatively high rate associated with elastic compression and consolidation to a slower rate of gradual creep. Assessment of post-creep behavior is appropriate for future studies. Typically the consolidation time was between 30 min and 1 h depending on the percentage of granulated rubber. For higher rubber percentage samples, more time was required for consolidation. High rubber content samples under high confining stress often experienced sufficient axial compression to cause the drive system magnets at the active end to come into contact with the bottoms of the coils of the torsional load system. This represented a total axial strain during consolidation on the order of 15%. This usually required adjustment of the level of coils for the 345 and 483 kPa confining pressures.

The ASTM D 4015 specification for resonant column testing was followed for procedures and data reduction. Typical shear modulus and damping ratios for specimens prepared by undercompaction at different rubber contents are shown in Figs. 1 to 4 for 29, 49, 76, and 100% rubber by volume, respectively, under different confinement pressures. Similar data were found for hand-spooned specimens. Figures 5a and 5b present the variation of shear modulus and damping ratio for different rubber contents at 345 kPa confinement. This shows that shear modulus decreases and damping ratio increases with increasing rubber content.

Modulus and Damping Characteristics of the Granulated Rubber/Sand Mixtures

Tire rubber is highly elastic. Therefore, the behavior of granulated rubber/sand mixtures will show a more elastic response with increasing rubber content. In Fig. 5a, the shear modulus degradation with increasing shear strain for 49% and 76% rubber samples is very small due to the high elasticity of rubber. Damping in a rubber/sand mixture is due to: (a) the friction of the particle contacts, and (b) the deformation of particles. The sand particles are very stiff and thus dissipate very little energy in particle deformation. In contrast, the rubber consumes energy through the deformation of rubber particles themselves. This is apparent in the 100% rubber sample in Fig. 4b. The figure shows that as the confinement pressure was increased, the damping ratio was found to increase slightly, which is the opposite of that found with typical soils.
Hardin (1978) developed an empirical equation of maximum shear modulus $G_{\text{max}}$ for particulate materials as:

$$G_{\text{max}} = \frac{A \cdot OCR^k}{F(e) - p_a^{1-n} \sigma_0^n}$$

(1)

where

- $A$ = dimensionless elastic stiffness parameter,
- $p_a$ = atmospheric pressure,
- $\sigma_0$ = effective mean principal stress,
- $F(e) = 0.3 + 0.7 e^2$, defining effect of void ratio on elastic stress-strain relationship,
- $e$ = void ratio,
- $OCR$ = overconsolidation ratio,
- $k$ = parameter related to soil plasticity index, and
- $n$ = elastic parameter.

Based on this equation, the $G_{\text{max}}$ values found in this study were normalized by dividing the $G_{\text{max}}$ values by $P_a^{0.5} \sigma_0^{0.5}$ with the estimation of $n = 0.5$ and $OCR = 1$. The results are shown in Fig. 6a for samples prepared by undercompaction and hand-spooning. The $G_{\text{max}}$ after normalization showed some scatter, probably because void ratio was not accounted for in the normalization. However, the scatter is minor because the major factor influencing the shear modulus is the percentage of rubber inclusion. In general, the maximum shear modulus for the samples prepared by undercompaction is a bit smaller than for those prepared by hand-spooning. This is partly due to the small mass of the water in the undercompaction samples which is moving with the particle frame. Hardin (1961) noted there was a reduction in the stiffness of the frame when water was added to a similar particle size of sand. Further, the samples
FIG. 6—$G_{\text{max}}/\left[\text{OCR}^3\sigma^*\right]$ and $D_{\text{min}}$ at 5e-4% shear strain versus percent rubber by volume for all specimens and confinement pressures.

FIG. 7—Back-calculated shear modulus from the triaxial tests by Masad et al. (1996) and the results of 49% rubber specimens.

FIG. 8—Normalized shear modulus degradation curves versus strain normalized by reference strain for 49% rubber specimens.
prepared by hand-spooning were observed to be less uniform than by undercompaction. Consequently, more sand clusters existed in the samples. The vibrational stress waves likely propagated faster in a sample with some clusters of sands than in a uniformly distributed mixture of rubber and sand. In other words, the rubber dominates the strength behavior in a uniform mixture, thus resulting in lower modulus.

In comparing the minimum damping ratio at a shear strain of $5 \times 10^{-4}$% in Fig. 6b, the trend of increasing damping with tire inclusion is consistent regardless of preparation method and is judged to be less related to the sample preparation method. It was not possible to determine $D_{\text{max}}$ in this study due to the large shear strains that would be required to achieve $D_{\text{max}}$.

**Reference Strain and Normalized Shear Modulus**

For soils, the normalized shear modulus reduction curves can be further simplified by using the reference strain concept (Hardin and Drnevich 1972). The reference strain ($\gamma_r$) is defined as $\gamma_r = \frac{\tau_{\text{max}}}{G_{\text{max}}}$, where $\tau_{\text{max}}$ is the shear stress at failure and depends on the initial state of stress and stress path. Data from a series of monotonic triaxial tests performed on a similar blend of granulated rubber and Ottawa sands (50% by volume) by Masad et al. (1996) was used to obtain the reference strain and the corresponding shear modulus at large strain. The shear modulus versus strain for the 200 and 350 kPa confinement can also be plotted along with the results from the resonant column tests as shown in Fig. 7. The failure envelope for the mixtures by Masad et al. (1996) can be plotted to obtain $\tau_{\text{max}}$ at different initial confinements. The $\tau_{\text{max}}$ were determined as 50, 122, 204, and 277 kPa for the isotropic confinements of 69, 207, 345, and 483 kPa, respectively. The normalized shear modulus is presented in Fig. 8 showing that the reference strain concept works well for rubber/soil mixtures too. The reduction curves collapsed very nicely for both the resonant column tests and the triaxial test data. This trend appears to be close to that for a typical saturated cohesive soil, shown in Fig. 8, which was generated using the empirical equation as indicated by Hardin and Drnevich (1972):

$$\frac{G}{G_{\text{max}}} = \frac{1}{1 + \frac{\gamma}{\gamma_r} \left[ 1 + a \cdot \exp \left( -b \cdot \frac{\gamma}{\gamma_r} \right) \right]}$$

where

- $a$, $b$ = soil constants,
- \(\exp\) = base of natural logarithms,
- $\gamma$ = shear strain, and
- $G$ = shear modulus.

It is interesting to see how the normalized curves for the granulated rubber/soil mixtures compared to the wheat samples tested by K. O. Hardin (1987). He suggested that $a = 1.3$ and $b = 0.1$ in Eq 2 for wheat. Figure 8 shows the mixture and wheat exhibit similar modulus reduction with increasing strain.

The modulus of rubber is very low relative to that of soil, hence the contribution of rubber to shear modulus in the mixture may not be significant. If so, the volume of rubber in a rubber/soil sample could be treated as voids; i.e., voids in the mixture include the volume of rubber and air. Equation 1 can then be used with the void ratio computed assuming rubber as voids to estimate the $G_{\text{max}}$ for the mixtures. The void ratio at 69 kPa, assuming rubber solids to be a part of the voids, was computed for each of the specimens. The value of $G_{\text{max}}/[OCR(P_u,\sigma_0)^{1.5}]$ versus this void ratio at 69 kPa confinement is shown in Fig. 9, along with the prediction of Eq 1 assuming $A = 600$ and $n = 0.5$. Therefore, it appears that the application of Eq 1 can be extended to the rubber/soil mixtures if the rubber is assumed to act as voids in the determination of $G_{\text{max}}$.

**Conclusions**

The paper presents resonant column test results for Ottawa sand mixed with granulated tire rubber. The tests were performed on materials prepared either air-dried or with a small quantity of moisture, using dry spooning or undercompaction to simulate typical in place fills. Small strain shear modulus and damping ratio are presented against shear strain, and the findings are normalized using models developed for soils with no rubber present.

The shear modulus of the mixtures is strongly influenced by the percentage of the rubber inclusion, as expected. Damping ratio increased slightly with confinement pressure for the 100% rubber, an opposite response from soil. This may be because under increasing confining stress, the size of interparticle contacts between particles increases significantly due to the presence of rubber. The reference strain concept is shown to be applicable to rubber/soil mixtures and the behavior of the mixture is identified as close to a cohesive soil and wheat. When the volume of rubber in a mixture is treated as voids, the $G_{\text{max}}$ can be estimated from Hardin’s empirical equation, that is, when given a known percentage of tire inclusion and air void for a rubber/soil mixture, the maximum shear modulus of the mixture can be estimated.

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**References**


